Time Series Trading Strategies Based on Almost Stochastic Dominances

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Abstract

This study investigates the application of almost stochastic dominance time-series trading strategies for cryptocurrency, specifically using 5-minute BTC returns. We propose a novel approach that compares current and past returns to determine investor preferences and inform trading positions. Our empirical analysis reveals that almost second-order stochastic dominance (ASSD) consistently outperforms other strategies, including buy-and-hold (BH), first-order stochastic dominance (FSD), and second-order stochastic dominance (SSD), across various time frequencies. These findings indicate that ASSD effectively captures investor behavior and preferences in the cryptocurrency market. Additionally, the strategies demonstrate varying performance in bull and bear markets, with SSD and ASSD offering better risk-adjusted returns during downturns. This research contributes to the stochastic dominance literature by highlighting the practical applications of ASD in time-series analysis and suggests new avenues for future research in investment strategy development.

Keyword: Almost Stochastic Dominances, Time series trading strategies, Cryptocurrency

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1. Introduction

Stochastic dominance (SD), as developed by Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970) and Whitmore (1970), is a decision-making framework for comparing two probability distributions to determine which is preferable, without relying on any specific investor preferences. However, SD criteria can sometimes fail to identify dominance, even when most reasonable decision-makers would clearly favor one investment over another. To address the limitations of SD, almost stochastic dominance (ASD) was introduced by Leshno and Levy (2002) and later refined by Levy (2012), Tzeng et al. (2013), Tsetlin et al. (2015) and Chang et al. (2019). ASD establishes dominance for all reasonable preferences while excluding extreme or pathological cases.

Numerous studies have explored applications of SD or ASD criteria. Post (2003) provided an empirical method for testing the SD efficiency of a given portfolio against all possible portfolios constructed from a set of assets, showing that the Fama and French market portfolio is significantly inefficient compared to benchmark portfolios based on market capitalization and book-to-market equity ratios. Regarding the common investment strategy of reallocating funds from stocks to bonds as investors age, Bali et al. (2009) demonstrated that the ASD criterion clearly supports advising a higher stock-to-bond ratio for long investment horizons. In stock index comparisons, Al-Khazali et al. (2014) used the SD criterion to investigate whether the Dow Jones Islamic indices outperform their conventional counterparts,[4] concluding that Islamic indices did outperform s during 2007–2012 global financial crisis. For the diversification role of gold in stock portfolios, Alkhazali and Zoubi (2020) employed the SD criterion to show that a gold-Islamic stock portfolio stochastically dominates one without gold across all Islamic stock indices. In comparing hedge funds, Bali et al. (2013) utilized the ASD criterion to assess whether hedge funds dominate the U.S. equity and bond markets, finding that long/short equity hedge and emerging markets hedge fund strategies outperform the U.S. equity

[4] Dow Jones conventional counterparts: Asia Pacific, Canadian, Developed Country, Emerging Markets, European, Global, Japanese, UK, and US indexes.

market, while long/short equity hedge, multistate, managed futures, and global macro hedge fund strategies dominate the U.S. Treasury market. More recently, Han et al. (2024) constructed 29 long–short cryptocurrency risk factor portfolios, as compiled by Liu et al. (2022) and Feng et al. (2020), and compared them with four benchmarks—S&P 500 Index, T-Bond, Bitcoin, and T-Bill—using the ASD criterion. They found that eight of the 29 portfolios could not dominate the four benchmarks. In the realm of abnormal returns in the stock market, Clark and Kassimatis (2014) generated abnormal returns using SD criterion, while Chiang et al. (2024) used ASD criterion. These abnormal returns demonstrate statistically and economically significant results when tested against alternative common risk factors.

Previous studies have primarily focused on using SD or ASD criterion to compare the return distributions of multiple assets or portfolios over the same period. In contrast, this paper introduces a novel approach by applying SD and ASD criteria to compare the return distributions of a single asset across two distinct time periods, thereby developing time-series-based investment strategies. The core concept of these time-series-based investment strategies by SD and ASD criteria is as follows: when comparing the current period returns of an asset with its past period returns, if the distribution of current period returns stochastically dominates that of past period returns, it indicates that investors prefer the current period returns over the past period returns. This preference is likely to increase (or decrease) demand for the asset, thereby driving its price up (or down), ceteris paribus. In accordance with this concept, we establish a long (or short) position in this asset for the subsequent period. Consequently, we establish a long (short) position in this asset for the next period. Conversely, when the distribution of current period returns does not stochastically dominate that of past period returns, and the distribution of past period returns does not stochastically dominate that of current returns, it suggests that investors cannot decisively compare the current and past returns. This indecision leads them to maintain their existing positions in the subsequent period.

We selected Bitcoin as the primary subject analysis for the SD and ASD time series strategies. Unlike traditional financial assets, Bitcoin's price is not backed by gold or any intrinsic value; instead, it relies solely on supply and demand. Since Bitcoin's supply is fixed, with a maximum limit of 21 million coins[5], its price movements are primarily driven by changes in demand. When investors anticipate that the price of Bitcoin will rise, they are more likely to buy and hold it, which increases demand and pushes the price higher. Conversely, if they expect the price to fall, they may sell Bitcoin, increasing market supply and driving the price down. Chen (2021) Chen (2021) verified that investor expectations are one of the primary factors affecting Bitcoin's price. The notion that Bitcoin's price is derived from investors' behavior based on their expectations aligns with the concept of our SD and ASD time series strategies, which captures how investors make decisions according to their preferences. Therefore, we have chosen Bitcoin as the focus of our analysis.

In the empirical analysis, we collect the 5-minute BTC returns from the Binance website[6] and compare the performance of five time series strategies: buy-and-hold (BH) strategy, first-order stochastic dominance (FSD) strategy, almost first-order stochastic dominance (AFSD) strategy, second-order stochastic dominance (SSD) strategy, and almost second-order stochastic dominance (ASSD) strategy. The empirical results indicate that, across all time frequencies (daily, weekly, monthly, quarterly, and yearly), the ASSD strategy exhibits the best performance regarding average excess returns and Sharpe ratios. Furthermore, the ASSD strategy outperforms the SSD strategy, while the AFSD strategy surpasses the FSD strategy in terms of average excess returns and Sharpe ratios. These findings suggest that the reasonable preferences underlying the ASD strategies align more closely with those of investors in the BTC market. Lastly, these strategies perform differently in bull and bear markets. While the BH strategy demonstrates strong returns in bull markets, it lacks resilience during bear markets. In contrast, the SSD and ASSD strategies offer better risk-adjusted returns, particularly in challenging bear market conditions.

[5] Bitcoin's open-source code was first released by Nakamoto (2008) and made available on GitHub and other version control platforms. The key rule regarding Bitcoin's supply cap is embedded in the block reward algorithm, which reduces the mining reward by half every 210,000 blocks, until the total supply reaches 21 million.

[6] Binance is one of the largest cryptocurrency exchanges in the world, founded in 2017. Known for its extensive range of crypto services, Binance provides spot trading, futures trading, staking, and decentralized finance (DeFi) products. https://www.binance.com/zh-TC

The remainder of the present study is structured as follows: Section 2 reviews ASD and develops ASD time series strategies; Section 3 provides an empirical example of ASD time series strategies on BTC; and Section 4 presents conclusions. The mathematical derivations are presented in the Appendix.

2. Time series strategies based on almost stochastic dominances

We first define the time series SDs and ASDs in section 2.1, and then develop their trading rules in section 2.2.

2.1. Time series of (almost) stochastic dominances

t denotes point in time, and $r_t \equiv (y_t - y_{t-1})/y_{t-1}$ denotes one period return of an asset earned during time period $[t-1, t]$, where y_t is its price at time t. The entire returns are divided into subsamples, and $\boldsymbol{R}_t$ represents the subsample return vector at time t collecting previous n returns in sequence: $\boldsymbol{R}_t \equiv \left(r_{t-(n-1)},\ r_{t-(n-2)},\ r_{t-(n-3)}, \dots,\ r_{t-2},\ r_{t-1},\ r_t\right)$. Let $g \in G$ to be possible outcome of $\boldsymbol{R}_t$ for all t, and $D_t(g)$ denotes the cumulative distribution of $\boldsymbol{R}_t$.

$D_t(g)$ dominates $D_{t-k}(g)$ by FSD if and only if $D_t(g) \leq D_{t-k}(g))$ for all $g \in G$ (with at least one strict inequality). AFSD of $D_t(g)$ over $D_{t-k}(g)$ means that $D_t(g) \leq D_{t-k}(g))$ for most of the range G, except for a relatively small segment that "violates" the dominance. $G^F(t, t-k) \equiv \{g\colon\ D_t(g) > D_{t-k}(g)\}$ denotes the range over which FSD is violated at time t, and $\bar{G}^F(t, t-k) \equiv \{g\colon\ D_t(g) \leq D_{t-k}(g))\}$ denotes the complement area of G. That is, $G^F(t, t-k) \cup \bar{G}^F(t, t-k) = G$. $v_t^F(t, t-k)$ is violation ratio defined as the cumulative distribution of $G^F(t, t-k)$ over that of G:

$$v_t^F(t, t-k) \equiv \frac{\int_{G^F(t,t-k)} \big(D_t(g) - D_{t-k}(g)\big) dg}{\int_G |D_t(g) - D_{t-k}(g)| dg}. \tag{1}$$

Definition 1 $D_t(g)$ is said to dominate $D_{t-k}(g)$ by $v_t^F(t, t-k)$-AFSD at time t when

$v_t^F(t, t-k) < a$, where $0 \leq a \leq 1$ is predetermined allowance violation ratio. The smaller $v_t^F(t, t-k)$, the stronger this dominance.

Leshno and Levy (2002) prove that if $D_t(g)$ dominates $D_{t-k}(g)$ by $v_t^F(t, t-k)$-AFSD, then the expected value of $\boldsymbol{R}_t$ is greater or equal to that of $\boldsymbol{R}_{t-k}$ for a class of preferences $u \in U_F^*\big(v_t^F(t, t-k)\big)$, where $U_F^*\big(v_t^F(t, t-k)\big)$ is the set of all "well-behaved" or "reasonable" non-decreasing utility functions, given by:

$$U_F^*\big(v_t^F(t, t-k)\big) = \left\{u: u'(x) > 0, u'(x) \leq \inf\{u'(x)\}\left[\frac{1}{v_t^F(t, t-k)}\right] \ \forall x \in G\right\}.$$

[Insert **Table 1** here]

Table 1 gives an example to generate a time series of AFSD through the returns, subsample return vector, cumulative distribution, and violation ratios for the case: $n = 3$ and $k = 2$. At time t, we collect subsample return vectors $\boldsymbol{R}_t = (r_{t-2},\ r_{t-1},\ r_t)$ and $\boldsymbol{R}_{t-2} = (r_{t-4},\ r_{t-3},\ r_{t-2})$, calculate their cumulative distributions $D_t(g)$ and $D_{t-2}(g)$, respectively, and then we calculate violation ratio $v_t^F(t, t-2)$ for AFD through Eq. (1). When $v_t^F(t, t-2) < a$, $D_t(g)$ dominates $D_{t-2}(g)$ by $v_t^F(t, t-2)$-AFSD, implying that the all investors with utility $U_F^*\big(v_t^F(t, t-k)\big)$ prefer $D_t(g)$ to $D_{t-2}(g)$. Similarly, we can calculate $v_{t+1}^F(t+1, t-1)$ $[v_{t+2}^F(t+2, t)]$ at time $t+1$ $[t+2]$ by collecting $\boldsymbol{R}_{t+1} = (r_{t-1},\ r_t,\ r_{t+1})$ and $\boldsymbol{R}_{t-1} = (r_{t-3},\ r_{t-2},\ r_{t-1})$ $[\boldsymbol{R}_{t+2} = (r_t,\ r_{t+1},\ r_{t+2})$ and $\boldsymbol{R}_t = (r_{t-2},\ r_{t-1},\ r_t)]$ and determine the AFSD between $D_{t+1}(g)$ and $D_{t-1}(g)$ $[D_{t+2}(g)$ and $D_t(g)]$. By analogy, we can generate a time series of AFSD decisions, determining at each time t whether $D_t(g)$ dominating $D_{t-2}(g)$ for the all investors with utility $U_F^*\big(v_t^F(t, t-2)\big)$.

Property 1 $D_t(g)$ is said to FSD dominates $D_{t-k}(g)$ at time t for $v_t^F(t, t-k) = a = 0$, and

$U_F^*\left(v_t^F(t,t-k)\right)$ coincides with the usual set of all non-decreasing utilities. In other words, AFSD reduces to the standard FSD criterion if there is no violation area at all.

$D_t(g)$ dominates $D_{t-k}(g)$ by SSD if and only if

$$\int_{-\infty}^{g} D_t(x)dx \le \int_{-\infty}^{g} D_{t-k}(x)dx$$

for all $g \in G$. Assume that the inequality holds for most of the range G, but not for all of it. Denote the area of SSD violation by:

$$G^S(t,t-k) \equiv \left\{g\colon\ D_t(g) > D_{t-k}(g);\ \int_{-\infty}^{g} D_{t-k}(x)dx \le \int_{-\infty}^{g} D_t(x)dx\ \right\}$$

and denote the complement area of $G^S(t,t-k)$ by $\bar{G}^S(t,t-k)$. That is $G^S(t,t-k) \cup \bar{G}^S(t,t-k) = G$. Define $v_t^S(t,t-k)$ as the ratio:

$$v_t^S(t,t-k) \equiv \frac{\int_{G^S(t,t-k)}\left(D_t(g) - D_{t-k}(g)\right)dg}{\int_G |D_t(g) - D_{t-k}(g)|dg}.$$

Definition 2 $D_t(g)$ is said to dominate $D_{t-k}(g)$ by $v_t^S(t,t-k)$-ASSD if $v_t^S(t,t-k) < a$. The smaller $v_t^S(t,t-k)$, the stronger this dominance.

Leshno and Levy (2002) prove that $D_t(g)$ dominating $D_{t-k}(g)$ by $v_t^S(t,t-k)$-ASSD implies that the investors with preferences $u \in U_S^*\left(v_t^S(t,t-k)\right)$ will prefer $D_t(g)$, where $u \in U_S^*\left(v_t^S(t,t-k)\right)$ is the set of all non-negative and concave utility functions, given by:

$$U_S^*\left(v_t^S(t,t-k)\right) = \left\{u\colon u''(x) \le \inf\{u''(x)\}\left[\frac{1}{v_t^S(t,t-k)}\right]\ \ \forall x \in G\right\}.$$

Property 2 $D_t(g)$ is said to dominate $D_{t-k}(g)$ by SSD at time t for $v_t^S(t,t-k) = a = 0$, and $U_S^*\left(v_t^F(t,t-k)\right)$ coincides with the usual set of all risk-averse utilities. In other words, ASSD reduces to the standard SSD criterion if there is no violation area at all.

Similar to the time series of AFSD decisions, we can also generate the time series of ASSD decisions, determining at each time t whether $D_t(g)$ dominating $D_{t-k}(g)$ for the all investors with utility $U_S^*\left(v_t^S(t,t-k)\right)$ through calculating $v_t^S(t,t-k)$ and comparing it with predetermined allowance violation ratio a.

2.2. Trading strategies

For simplicity, $G(t,t-k)$ represents $G^F(t,t-k)$ or $G^S(t,t-k)$, $\bar{G}(t,t-k)$ represents $\bar{G}^F(t,t-k)$ or $\bar{G}^S(t,t-k)$, and then $G = G(t,t-k) \cup \bar{G}(t,t-k)$ represents $G^F(t,t-k) \cup \bar{G}^F(t,t-k)$ or $G^S(t,t-k) \cup \bar{G}^S(t,t-k)$. Furthermore, $v_t\ (t,t-k)$ denoting $v_t^F(t,t-k)$ or $v_t^S(t,t-k)$ can be written as

$$v_t\ (t,t-k) \equiv \frac{\int_{G(t,t-k)}\left(D_t(g) - D_{t-k}(g)\right)dg}{\int_G |D_t(g) - D_{t-k}(g)|dg}. \tag{1}$$

Furthermore, $U^*\left(v_t\ (t,t-k)\right)$ denotes $U_F^*\left(v_t^F(t,t-k)\right)$ or $U_S^*\left(v_t^S(t,t-k)\right)$.

2.2.1. ASD trading strategies

Before formally defining the trading rules of ASD strategies, we first provide a useful proposition as follows.

Proposition 1 Given an allowance violation ratio $0 < a < 1$, we have the following properties.

(i). For $v_t\ (t,t-k) < a$, $D_t(g)$ dominates $D_{t-k}(g)$ by $v_t\ (t,t-k)$-ASD, and the smaller $v_t\ (t,t-k)$, the stronger this dominance.

(ii). For $v_t\ (t,t-k) > 1-a$, $D_{t-k}(g)$ dominates $D_t(g)$ by $v_t\ (t,t-k)$-ASD, and the larger $v_t\ (t,t-k)$, the stronger this dominance.

(iii). For $a \le v_t\ (t, t-k) \le 1-a$, $D_t(g)\ [D_{t-k}(g)]$ does not dominate $D_{t-k}(g)\ [D_t(g)]$ by $v_t\ (t, t-k)$-ASD.

Proof: Please see **Appendix A**.

Given an allowance violation ratio $0 < a < 1$, while the time series of AFSD or ASSD only determine at each time t whether $D_t(g)$ dominating $D_{t-k}(g)$ by comparing $v_t\ (t, t-k)$ and a, **Proposition 1** enables us to determine at each time t both whether $D_t(g)$ dominating $D_{t-k}(g)$ and whether $D_{t-k}(g)$ dominating $D_t(g)$. This property help us to develop the following trading rules of ASD strategies.

ASD trading rules $P_t\ (a)$ denotes the position on the asset at time t given an allowance violation ratio a, and the ASD trading rules are constructed as follows.

(i). If $v_t\ (t, t-k) < a$, $P_{t+1}(a) = 1$.

According to (i) of **Proposition 1**, $v_t\ (t, t-k) < a$ indicates that $D_t(g)$ dominates $D_{t-k}(g)$ by $v_t\ (t, t-k)$-ASD, i.e., the utilities on the $\boldsymbol{R}_t$ are *greater* than those on the $\boldsymbol{R}_{t-k}$ for all the investors with $U^*\left(v_t\ (t, t-k)\right)$. This leads to an *increase in demand* for this asset, and consequently, its price will rise, ceteris paribus. Thus, we establish one long positions on this asset at time $t+1$.

(ii). If $v_t\ (t, t-k) > a$, $P_{t+1}(a) = -1$.

According to (ii) of **Proposition 1**, $v_t\ (t, t-k) > a$ indicates that $D_t(g)$ dominates $D_{t-k}(g)$ by $v_t\ (t, t-k)$-ASD, i.e., the utilities on the $\boldsymbol{R}_t$ are *smaller* than those on the $\boldsymbol{R}_{t-k}$ for all the investors with $U^*\left(v_t\ (t, t-k)\right)$. This leads to an *decrease in demand* for this asset, and consequently, its price will fall, ceteris paribus. Thus, we establish one short position on this asset at time $t+1$.

(iii). If $a \le v_t\ (t, t-k) \le 1-a,\ P_{t+1}(a) = 1$ *when* $P_t\ (a) = 1;\ P_{t+1}(a) = 0$ *when* $P_t\ (a) = 0;\ P_{t+1}(a) = -1$ *when* $P_t\ (a) = -1$.

The situation, $a \le v_t\ (t, t-k) \le 1-a$, implies that there is no $v_t\ (t, t-k)$-ASD between $D_t(g)$ and $D_{t-k}(g)$, i.e., all the investors with $U^*\left(v_t\ (t, t-k)\right)$ cannot determine whether $\boldsymbol{R}_t$ or $\boldsymbol{R}_{t-k}$ provides them higher utility, so maintain the previous position.

[Insert **Figure 1** here]

In **Figure 1**, we provides an example of determining position $P_t\ (a)$ through the violation ratio $v_t\ (t, t-k)$ and a predetermined allowance violation ratio a based on ASD trading rules (i), (ii), and (iii). In this example, the assumed violation ratios are shown in the top figure, while the corresponding positions are displayed in the bottom figure. We also assume that $P_0\ (a) = 0$.

At $t = 1$ and 2, we can observe that $v_1\ (1, 1-k)$ and $v_2\ (2, 2-k)$ are both smaller than a, and thus we have $P_2\ (a) = P_3\ (a) = 1$ based on ASD trading rule (i). At $t = 3$, because $a \le v_3\ (3, 3-k) \le 1-a$ and $P_3\ (a) = 1$, ASD trading rule (iii) indicates that $P_4\ (a) = 1$. Similarly, at $t = 4$, because $a \le v_4\ (4, 4-k) \le 1-a$ and $P_4\ (a) = 1$, ASD trading rule (iii) also indicates that $P_5\ (a) = 1$. Next, $v_5\ (5, 5-k)$ and $v_6\ (6, 6-k)$ are both larger than $(1-a)$, so ASD trading rule (ii) generates $P_6\ (a) = P_7\ (a) = -1$.

Furthermore, because $a \le v_7\ (7, 7-k) \le 1-a$ and $P_7\ (a) = -1$, ASD trading rule (iii) generates $P_8\ (a) = -1$, and similarly, because $a \le v_8\ (8, 8-k) \le 1-a$ and $P_8\ (a) = -1$, ASD trading rule (iii) also generates $P_9\ (a) = -1$. At $t = 9$ and 10, both $v_9\ (9, 9-k)$ and $v_{10}(10, 10-k)$ are smaller than a, and thus $P_{10}(a) = P_{11}(a) = 1$ based on ASD trading rule (i). Because $P_{11}(a) = 1$ and both $v_{11}(11, 11-k)$ and $v_{12}(12, 12-k)$ are between a and $1-a$, $P_{12}(a) = P_{13}(a) = -1$ based on ASD trading rule (iii). Final, according to ASD trading rule (ii),

$P_{14}(a) = 1$ because $v_{13}(13, 13 - k) > 1 - a$.

2.2.2. Trading rules of SD strategies

When $a = 0$, **Property 1** and **Property 2** indicate that $v_t\ (t, t-k) = 0$ implies $D_t(g)$ dominates $D_{t-k}(g)$ by SD. Similar to the time series of ASD, the time series of SD only determine at each time t whether $D_t(g)$ dominates $D_{t-k}(g)$ but cannot confirm whether $D_{t-k}(g)$ dominates $D_t(g)$. **Proposition 2** addresses this issue.

Proposition 2 Given $a = 0$, we have the following properties.

(i). For $v_t\ (t, t-k) = 0$, $D_t(g)$ dominates $D_{t-k}(g)$ by SD.

(ii). For $v_t\ (t, t-k) = 1$, $D_{t-k}(g)$ dominates $D_t(g)$ by SD.

(iii). For $0 < v_t\ (t, t-k) < 1$, $D_t(g)$ $[D_{t-k}(g)]$ does not dominate $D_{t-k}(g)$ $[D_t(g)]$ by SD

Proof: Please see **Appendix B**.

Proposition 2 enables us to determine at each time t both whether $D_t(g)$ dominates $D_{t-k}(g)$ and whether $D_{t-k}(g)$ dominating $D_t(g)$ by SD, and thus we have following SD trading rules (SDTRs).

SD trading rules $P_t\ (0)$ denotes the position on the asset at time t, and the SD trading rules are constructed as follows.

(i). If $v_t\ (t, t-k) = 0$, $P_{t+1}(0) = 1$.

According to (i) of **Proposition 2**, $v_t\ (t, t-k) = 0$ indicates that $D_t(g)$ dominates $D_{t-k}(g)$ by SD, i.e., the utilities on the $\boldsymbol{R}_t$ are *greater* than those on the $\boldsymbol{R}_{t-k}$ for all non-decreasing utility or all risk-averse investors. This leads to an *increase in demand* for this asset, and consequently, its price will rise, ceteris paribus. Thus, we establish one long positions on this asset at time $t+1$.

(ii). If $v_t\ (t, t-k) = 1,\ P_{t+1}(0) = -1.$

According to (ii) of **Proposition 2**, $v_t\ (t, t-k) = 1$ indicates that $D_t(g)$ dominates $D_{t-k}(g)$ by SD, i.e., the utilities on the $\boldsymbol{R}_t$ are smaller than those on the $\boldsymbol{R}_{t-k}$ for all non-decreasing utility or all risk-averse investors. This leads to an decrease in demand for this asset, and consequently, its price will fall, ceteris paribus. Thus, we establish one short position on this asset at time $t+1$.

(iii). If $0 < v_t\ (t, t-k) < 1,\ P_{t+1}(0) = 1$ *when* $P_t\ (0) = 1$*;* $P_{t+1}(0) = 0$ *when* $P_t\ (0) = 0$*;* $P_{t+1}(0) = -1$ *when* $P_t\ (0) = -1.$

In this situation, there is no SD between $D_t(g)$ and $D_{t-k}(g)$, i.e., all non-decreasing utility or all risk-averse investors cannot determine whether $\boldsymbol{R}_t$ or $\boldsymbol{R}_{t-k}$ provides them higher utility, so maintain the previous position.

[Insert **Figure 2** here]

In **Figure 2**, we provide an example of determining position $P_t\ (0)$ through the violation ratio $v_t\ (t, t-k)$ based on SD trading rules (i), (ii), and (iii). In this example, the assumed violation ratios are shown in the top figure, while the corresponding positions are displayed in the bottom figure. We also assume that $P_0\ (0) = 0$.

At $t = 1$ and 2, since $v_1\ (1, 1-k)$ and $v_2\ (2, 2-k)$ are both equal to 0, we have $P_2\ (0) = P_3\ (0) = 1$ based on SD trading rule (i). At $t = 3$ and 4, because $0 < v_3\ (3, 3-k) < 1$ and $0 < v_4\ (4, 4-k) < 1$ and $P_3\ (0) = 1$, SD trading rule (iii) indicates that $P_4\ (0) = P_5\ (0) = 1$. Furthermore, At $t = 5$ and 6, both $v_5\ (5, 5-k)$ and $v_6\ (6, 6-k)$ equal 1, so SD trading rule (ii) generates $P_6\ (0) = P_7\ (0) = -1$. Because $0 < v_7\ (7, 7-k) < 1$, SD trading rule (iii) generates $P_8\ (0) = -1$, and similarly, because $0 < v_8\ (8, 8-k) < 1$, SD trading rule (iii) also generates $P_9\ (0) = -1$. At $t = 9$ and 10, both $v_9\ (9{,}9 - k)$ and $v_{10}(10, 10 - k)$ equal 0, and thus $\mathrm{P}_{10}(0) = \mathrm{P}_{11}(0) = 1$ based on SD trading rule (i). Because $P_{11}(0) = 1$ and $0 < v_{11}(11, 11 - k) < 1$ and $0 < v_{12}(12, 12 - k) < 1$, $P_{12}(0) = P_{13}(0) = -1$ based on SD trading rule (iii). Final, according to SD trading rule (ii), $P_{14}(0) = 1$ because $v_{13}(13, 13 - k) = 1$.

3. Empirical analyses

This section demonstrates the applicability of the proposed SD and ASD time series strategies to BTC. We collect intraday 5-minute BTC returns from Binance and daily federal funds effective rates from the Federal Reserve Economic Data (FRED).[7] To synchronize the data frequency between BTC prices and the federal funds effective rates, we assume the federal funds effective rates remain constant throughout the day, with the 5-minute rate being equal to the daily rate divided by 288. **Table 2** represents the descriptive statistics of 5-min BTC returns, the federal funds effective rates from January 1, 2018, 00:00:00 to December 1, 2023, 23:55:00, and the total size for 5-min BTC returns or federal funds effective rates is 631,008. First, the standard deviation for 5-min BTC reruns is 2.41×10^{-3} , showing significant volatility, Second, the skewness of 5-min BTC returns is 6.04×10^{-1}, suggesting a positive skew, meaning that extreme positive returns are more common than extreme negative ones. Final, 5- min BTC returns have a kurtosis coefficient of 1.34×10^{2}, indicating extreme leptokurtic behavior, with lots of sharp price movements. These statistical characteristics shows the difference of cryptocurrency market and financial market.

[Insert **Table 3** here]

Next, we adopt five trading strategies on Bitcoin data: the buy-and-hold (BH) strategy, the FSD strategy, the AFSD strategy with $a = 0.06$ [referred to as the AFSD (0.06) strategy], the SSD

[7] FRED: https://fred.stlouisfed.org/

strategy, and the ASSD strategy with $a = 0.06$ [referred to as the ASSD (0.06) strategy]. The predetermined allowance violation ratio a is set as 0.06 for ASD strategies by following the experimentally estimate result of Levy et al. (2010). They find that if the violation ratio is smaller than this value, all subjects in their experiments selecting the ASD dominating investment. The latter studies such as Levy (2012) also follow this value for a. The subsample return vector $\boldsymbol{R}_t$ collects previous n 5-min BTC returns in sequence, where n is considered as 8640 (30 days) or 17280 (60 days) in this study. By using these subsample return vectors, we calculate the violation ratio $v_t\ (t, t-k)$ for SD and ASD strategies, where k is set as 288 (1 days).

Let $P_t \in \{P_t^B,\ P_t^F(0), P_t^F(0.06), P_t^S(0), P_t^S(0.06)\}$, where P_t^B, $P_t^F(0)$, $P_t^F(0.06)$, $P_t^S(0)$, $P_t^B(0.06)$ denotes the positions of the BH strategy, the FSD strategy, the AFSD (0.06) strategy, the SSD strategy, and the ASSD (0.06) strategy at time t. The strategy excess return at time t is calculated as $ER_t = P_{t-1} \times r_t - r_t^f$, where r_t^f is the federal funds effective rate at time t. The first violation ratios $v_t\ (t, t-k)$ of ASD strategies are established at December, 31, 2017, 23:50:00, and thus the first positions of these five strategies are established at December, 31, 2017, 23:55:00. Therefore, the excess returns of these five strategies are all established spanning from January 1, 2018, 00:00:00 to December 1, 2023, 23:55:00. In other words, we set the total evaluation period consists of 631,008 time points.

3.1. Periodic trading performance

This section compares the performances of excess returns among these five strategies periodically (daily, weekly, monthly, quarterly and yearly), and the results are reported in **Table 3**. T represents the total evaluation period and is divided into J sub-periods. The jth sub-period consists of T_j time points, i.e., $T = \sum_{j=1}^{J} T_j$. The periodic excess return for the jth sub-period is calculated as

$$PER_j = \prod_{i=1}^{T_j} (1 + ER_i) - 1.$$

The average value, standard deviation, and Sharpe ratio of $\boldsymbol{PER} = \{PER_j \mid j = 1, 2, 3, \dots, J\}$ are calculated by

$$AV(\boldsymbol{PER}) = \frac{\sum_{j=1}^{J} PER_j}{J},$$

$$SD(\boldsymbol{PER}) = \sqrt{\frac{\sum_{j=1}^{J}\left(PER_j - EV(\boldsymbol{PER})\right)^2}{J-1}},$$

and

$$SR(\boldsymbol{PER}) = \frac{AV(\boldsymbol{PER})}{SD(\boldsymbol{PER})}.$$

[Insert **Table 3** here]

Panel A and **Panel B** in **Table 3** reports the results for the window size of subsample n =8640 (30 days) and 17280 (60 days), respectively. In the daily frequency, the ASSD (0.06) strategy consistently exhibits the best performance across both 30-day and 60-day windows, yielding the highest Sharpe ratios of 0.0406 and 0.0501, respectively. This indicates that the ASSD (0.06) strategy offers the most favorable risk-adjusted returns in the short-term horizon. In contrast, the FSD strategy performs the weakest, particularly in the 30-day window with negative excess returns and Sharpe ratios, indicating unfavorable performance. The AFSD (0.06) strategy, while improving slightly in the 60-day window, underperforms compared to both the BH and SSD strategies. For weekly frequency, the ASSD (0.06) strategy once again demonstrates superior performance with the highest Sharpe ratios (0.1088 and 0.1346) across both 30-day and 60-day windows. The SSD strategy performs well regarding risk-adjusted returns but falls behind the ASSD (0.06) strategy. The BH strategy consistently remains in the middle range, while the FSD strategy underperforms in the 30-day window but shows improvement in the 60-day window.

In monthly observations, the ASSD (0.06) strategy continues to outperform, achieving the highest excess return and Sharpe ratio in both periods. The BH strategy follows closely but falls short in terms of risk-adjusted returns. The FSD strategy, which is notably the weakest in the 30-day window, improves significantly in the 60-day period, reflecting a pattern of better performance with longer holding periods. The AFSD (0.06) strategy shows improvement over the FSD, yet it remains behind the SSD and ASSD (0.06) strategies. The quarterly performance results align with earlier patterns, where the ASSD strategy leads with the highest Sharpe ratios (0.3741 and 0.4345). Interestingly, the FSD strategy underperforms in both time frames, indicating that this strategy is less suitable for longer-term periods. The SSD strategy, while showing decent returns, is overshadowed by the ASSD (0.06) strategy's consistently higher risk-adjusted performance. In the yearly horizon, the ASSD (0.06) strategy again ranks the highest with the best Sharpe ratios, confirming its dominance across all time frequencies. The FSD strategy, while underperforming at shorter time horizons, shows significant improvement with longer evaluation periods. However, it is still outperformed by the ASSD (0.06) and AFSD (0.06) strategies. The BH strategy remains competitive, but it cannot match the higher risk-adjusted returns of the ASSD (0.06) strategy.

Additionally, across all time frequencies (daily, weekly, monthly, quarterly, and yearly), the ASSD (0.06) strategy outperforms the SSD strategy while the AFSD (0.06) strategy outperforms the FSD strategy in terms of excess returns and Sharpe ratios. This result seems to reflect that ASD, considering all reasonable preferences, excluding extreme or pathological cases, are more consistent with investor choices in BTC market.

3.2. Trading performance during bull and bear markets

Following the bull/bear market definition in Lunde and Timmermann (2004), we set the threshold filter, (λ_1, λ_2) = (20, 20)[8], to find out the period of the bull and bear markets. Then, we divided the entire evaluation period into 5 bull market periods and 5 bear market periods, respectively.

[8] The threshold filter setting is the same as that used by Zhang et al. (2020) to distinguish the bull and bear periods in the cryptocurrency market.

The detail periods for each bull or bear market are listed in **Table 4**.

J^{Bull} (J^{Bear}) denotes the numbers of the bull (bear) market periods, and T_j^{Bull} (T_j^{Bear}) denotes the numbers of 5-min excess returns generated by each strategy in jth bull (bear) market periods. Thus, $T = \sum_{j=1}^{J^{Bull}} T_j^{Bull} + \sum_{j=1}^{J^{Bear}} T_j^{Bear}$. Let $M \in \{Bull, Bear\}$, and jth period excess return in bull or bear market is calculated as

$$PER_j = \prod_{i=1}^{T_j^M} (1 + ER_i) - 1.$$

The annualized period excess return for PER_j is calculated as

$$AER_j = \left(1 + PER_j\right)^{\frac{1}{Y_j^M}} - 1,$$

where Y_j^M is the corresponding number of year for jth bull or bear market periods. The average value, standard deviation, and Sharpe ratio of $\boldsymbol{AER} = \left\{AER_j \mid j = 1, 2, 3, \dots, J^M\right\}$ are calculated by

$$AV(\boldsymbol{AER}) = \frac{\sum_{j=1}^{J^M} AER_j}{J^M},$$

$$SD(\boldsymbol{AER}) = \sqrt{\frac{\sum_{j=1}^{J^M} \left(AER_j - EV(\boldsymbol{AER})\right)^2}{J^M - 1}},$$

and

$$SR(\boldsymbol{AER}) = \frac{AV(\boldsymbol{AER})}{SD(\boldsymbol{AER})}.$$

[Insert **Table 5** here]

Panel A and **Panel B** in **Table 5** reports the performance results during bull and bear markets and for the window size of subsample n=8640 (30 days) and 17280 (60 days), respectively. Regarding the bull market, the BH strategy consistently shows strong performance across both window sizes, with an average annualized return of 3.7645 and a Sharpe ratio of 1.3383, indicating solid returns with moderate risk. FSD Strategy initially underperforms in the 30-day period with an average return of -0.2222 but recovers to 1.3570 in the 60-day period, achieving a Sharpe ratio of 0.7619, reflecting improved risk-adjusted performance when window size increases. AFSD (0.06) strategy demonstrates positive returns in both periods (1.8099 in 30 days and 2.3005 in 60 days), with increasing Sharpe ratios from 0.4287 to 0.5765, suggesting better relative performance as the when window size extends. SSD strategy shows weak performance in the 30-day period but improves in the 60-day period. ASSD (0.06) stands out in both window size with the highest average returns (1.1454 and 0.4312) and high Sharpe ratios, indicating strong performance relative to risk.

Regarding bear market, BH strategy underperforms with negative returns (-0.7312) in both sample size and a very low Sharpe ratio (-4.8223 in 60 days), highlighting its vulnerability in bear markets. While still negative, FSD strategy returns improve from -0.0755 (30 days) to -0.1715 (60 days), but Sharpe ratios remain low, indicating continued challenges in bear markets. AFSD (0.06) shows a notable decline in performance with average returns dropping significantly from -0.5206 to -2.4114, illustrating sensitivity to market conditions. Contrarily, the SSD strategy performs well in bear markets with positive average returns (0.7153 in 30 days and 1.2671 in 60 days) and increasing Sharpe ratios, suggesting effectiveness in downturns. ASSD (0.06) strategy excels in bear markets with the highest average returns across both periods (2.5268 and 1.2069) and strong Sharpe ratios, showcasing resilience and effective risk management.

In summary, the 60-day window size case generally results in better performance metrics for most strategies, indicating that a longer window size can mitigate short-term volatility. Additionally, these strategies perform differently in bull and bear markets. While BH has strong returns in bull markets, it lacks resilience in bear markets. Conversely, SSD and ASSD strategies appear to offer better risk-adjusted returns, particularly in challenging bear market conditions.

4. Conclusion

This study explores the application of SD and ASD to develop time-series trading strategies based on the comparative analysis of current and past returns of a single asset. Our empirical findings reveal that the ASSD strategy outperforms other strategies across various time frequencies, including FSD, AFSD, and SSD. This superior performance, particularly in terms of average excess returns and Sharpe ratios, underscores the relevance of the ASD framework in capturing the preferences of investors. Additionally, the ability of AFSD (ASSD) to consistently outperform FSD (SSD) suggests that incorporating ASD can yield significant insights into market dynamics and investor behavior, particularly in a volatile asset class like cryptocurrency.

Moreover, the differentiated performance of the strategies in bull and bear markets highlights the importance of tailoring investment approaches to prevailing market conditions. While the buy-and-hold (BH) strategy may thrive in upward trends, it lacks resilience in downturns. Conversely, the SSD and ASSD strategies demonstrate better risk-adjusted returns, making them more suitable for investors navigating turbulent market environments.

Overall, this study contributes to the existing literature on SD and ASD and opens new avenues for research by demonstrating the applicability of SD and ASD in time-series contexts. Future research could further refine these strategies or explore their applicability across other asset classes, enhancing our understanding of investor preferences and market behavior. By integrating advanced decision-making frameworks like ASD, investors can potentially make more informed and strategic decisions, ultimately improving investment outcomes in an increasingly complex financial landscape.

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Appendix

Appendix A

This appendix derives **Proposition 1**. (i) of **Proposition 1** summarizes **Definition 1** and **Definition 2**. (ii) and (iii) of **Proposition 1** are derived as follows.

Firstly, let us to derive that $v_t\ (t, t-k) + v_t\ (t-k, t) = 1$. Recalling Eq. (1) yields

$$v_t\ (t, t-k) \equiv \frac{\int_{G(t,t-k)} \left(D_t(g) - D_{t-k}(g)\right) dg}{\int_G |D_t(g) - D_{t-k}(g)| dg},$$

which can be rewritten as

$$v_t\ (t, t-k) \equiv \frac{\int_{G(t,t-k)} |D_t(g) - D_{t-k}(g)| dg}{\int_G |D_t(g) - D_{t-k}(g)| dg}$$

because $D_{n+1}(g) - D_n(g) > 0$ on $G(t, t-k)$. Following the previous equation, we can calculate $v_t\ (t-k, t)$ as

$$v_t\ (t-k, t) = \frac{\int_{G(t-k,t)} |D_{t-k}(g) - D_t(g)| dg}{\int_G |D_{t-k}(g) - D_t(g)| dg} = \frac{\int_{G(t-k,t)} |D_t(g) - D_{t-k}(g)| dg}{\int_G |D_t(g) - D_{t-k}(g)| dg}.$$

Because $G = G(t, t-k) \cup \bar{G}(t, t-k)$, the sum of the previous two equations is therefore

$$v_t\ (t, t-k) + v_t\ (t-k, t)$$

$$= \frac{\int_{G(t,t-k)} |D_t(g) - D_{t-k}(g)| dg + \int_{G(t-k,t)} |D_t(g) - D_{t-k}(g)| dg}{\int_G |D_t(g) - D_{t-k}(g)| dg} = \frac{\int_G |D_t(g) - D_{t-k}(g)| dg}{\int_G |D_t(g) - D_{t-k}(g)| dg} = 1.$$

Next, we prove (ii) of **Proposition 1**. $v_t\ (t-k, t) = 1 - v_t\ (t, t-k)$ implies that $v_t\ (t-k, t) < a$ is equivalent to $v_t\ (t-k, t) > 1 - a$. Thus, the fact, " $D_{t-k}(g)$ dominates $D_t(g)$ by $v_t\ (t-k, t)$-ASD for $v_t\ (t-k, t) < a$; the smaller $v_t\ (t-k, t)$, the stronger this dominance. ", is equivalent to that " $D_{t-k}(g)$ dominates $D_t(g)$ by $v_t\ (t, t-k)$-ASD for $v_t\ (t, t-k) > 1 - a$; the larger $v_t\ (t, t-k)$, the stronger this dominance. ".

Final, we prove (iii) of **Proposition 1**. (i) of **Proposition 1** implies that $D_t(g)$ does not dominate $D_{t-k}(g)$ by $v_t\ (t, t-k)$-ASD for $v_t\ (t, t-k) \geq a$. Furthermore, (ii) of **Proposition 1** implies that $D_{t-k}(g)$ does not dominate $D_t(g)$ by $v_t\ (t, t-k)$-ASD for $v_t\ (t, t-k) \leq 1-a$ Thus, for $a \leq v_t\ (t, t-k) \leq 1-a$, $D_t(g)$ $[D_{t-k}(g)]$ does not dominate $D_{t-k}(g)$ $[D_t(g)]$ by $v_t\ (t, t-k)$-ASD. The proof is complete.

Appendix B

This appendix derives **Proposition 2**. Part (i) of **Proposition 2** summarizes **Property 1** and **Property 2**. Parts (ii) and (iii) of **Proposition 2** are derived as follows.

Applying $v_t\ (t-k, t) = 1 - v_t\ (t, t-k)$ derived in **Appendix A**, we have that $v_t\ (t-k, t) = 0$ is equivalent to $v_t\ (t-k, t) = 1$. Thus, the fact, " $D_{t-k}(g)$ dominates $D_t(g)$ by SD when $v_t\ (t-k, t) = 0$ " is equivalent to that " $D_{t-k}(g)$ dominates $D_t(g)$ by SD when $v_t\ (t-k, t) = 1$". The proof of Parts (ii) is complete.

Part (i) of **Proposition 2** implies that $D_t(g)$ does not dominate $D_{t-k}(g)$ by SD when $v_t\ (t, t-k) > 0$. Further, Part (ii) of **Proposition 2** implies that $D_{t-k}(g)$ does not dominate $D_t(g)$ by SD for $v_t\ (t, t-k) < 1$ Thus, for $0 < v_t\ (t, t-k) < 1$, $D_t(g)$ $[D_{t-k}(g)]$ does not dominate $D_{t-k}(g)$ $[D_t(g)]$ by SD. The proof of Part (iii) is complete.

Tables

Table 1: Return, subsample return vector, cumulative distribution, and violation ratio for $n = 3$ and $k = 2$.

Time in point							Subsample	Cumulative	Violation
$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	return vector	distribution	ratio
r_{t-4}	r_{t-3}	r_{t-2}					$\boldsymbol{R}_{t-2}$	$D_{t-2}(g)$	$v_t^F(t, t-2)$
		r_{t-2}	r_{t-1}	r_t			$\boldsymbol{R}_t$	$D_t(g)$	
	r_{t-3}	r_{t-2}	r_{t-1}				$\boldsymbol{R}_{t-1}$	$D_{t-1}(g)$	$v_{t+1}^F(t+1, t-1)$
			r_{t-1}	r_t	r_{t+1}		$\boldsymbol{R}_{t+1}$	$D_{t+1}(g)$	
		r_{t-2}	r_{t-1}	r_t			$\boldsymbol{R}_t$	$D_t(g)$	$v_{t+2}^F(t+2, t)$
				r_t	r_{t+1}	r_{t+2}	$\boldsymbol{R}_{t+2}$	$D_{t+2}(g)$	

Note: This table represents the relationship among returns, subsample returns, cumulative distributions, and violation ratios for the case: $n = 3$ and $k = 2$. At time t, we collect subsample returns $\boldsymbol{R}_t = (r_{t-2}, r_{t-1}, r_t)$ and $\boldsymbol{R}_{t-2} = (r_{t-4}, r_{t-3}, r_{t-2})$, calculate their cumulative distributions $D_t(g)$ and $D_{t-k}(g)$, respectively, and then calculate violation ratio $v_t^F(t, t-2)$ through Eq. (1). Similarly, we can calculate $v_{t+1}^F(t+1, t-1)$ $[v_{t+2}^F(t+2, t)]$ at time $t+1$ $[t+2]$ through $\boldsymbol{R}_{t+1} = (r_{t-1}, r_t, r_{t+1})$ and $\boldsymbol{R}_{t-1} = (r_{t-3}, r_{t-2}, r_{t-1})$ $[\boldsymbol{R}_{t+2} = (r_t, r_{t+1}, r_{t+2})$ and $\boldsymbol{R}_t = (r_{t-2}, r_{t-1}, r_t)]$.

Table 2: Descriptive statistics of 5-min BTC returns and federal funds effective rates

	BTC returns	**Federal funds effective rates**
Average return	4.71×10^{-6}	1.77×10^{-7}
Standard deviation	2.41×10^{-3}	1.66×10^{-7}
Skewness coefficient	6.04×10^{-1}	7.21×10^{-1}
Kurtosis coefficient	1.34×10^{2}	-6.05×10^{1}
Minimum	-1.02×10^{-1}	3.81×10^{-9}
Maximum	1.84×10^{1}	5.07×10^{-7}
Sample size	631,008	631,008

Note: This table reports the descriptive statistics of 5-min BTC returns and federal funds effective rates, including average return, standard deviation, skewness coefficient, kurtosis coefficient, minimum value, maximum value, and sample size.

Table 3: Periodic strategy performance

Panel A: n =8640 (30 days)

Daily

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.0012 (4)	0.0380 (5)	0.0314 (4)
FSD	-0.0003 (1)	0.0307 (1)	-0.0090 (1)
AFSD (0.06)	-0.0001 (2)	0.0375 (4)	-0.0028 (2)
SSD	0.0011 (3)	0.0369 (2)	0.0298 (3)
ASSD (0.06)	0.0015 (5)	0.0371 (3)	0.0406 (5)

Weekly

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.0081 (4)	0.0974 (5)	0.0834 (4)
FSD	-0.0020 (1)	0.0801 (1)	-0.0250 (1)
AFSD (0.06)	-0.0009 (2)	0.0973 (4)	-0.0096 (2)
SSD	0.0070 (3)	0.0901 (2)	0.0774 (3)
ASSD (0.06)	0.0099 (5)	0.0913 (3)	0.1088 (5)

Monthly

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.0365 (4)	0.2153 (5)	0.1698 (4)
FSD	-0.0075 (1)	0.1713 (1)	-0.0437 (1)
AFSD (0.06)	-0.0045 (2)	0.1997 (4)	-0.0227 (2)
SSD	0.0285 (3)	0.1825 (3)	0.1564 (3)
ASSD (0.06)	0.0405 (5)	0.1776 (2)	0.2283 (5)

Quarterly

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.1655 (5)	0.5973 (5)	0.2771 (4)
FSD	0.0104 (1)	0.4751 (3)	0.0220 (1)
AFSD (0.06)	0.0392 (2)	0.5355 (4)	0.0733 (2)
SSD	0.0992 (3)	0.4125 (2)	0.2406 (3)
ASSD (0.06)	0.1431 (4)	0.3825 (1)	0.3741 (5)

Yearly

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.7676 (4)	1.4064 (4)	0.5458 (4)
FSD	-0.1500 (1)	0.3686 (1)	-0.4070 (1)
AFSD (0.06)	-0.0515 (2)	0.6994 (2)	-0.0736 (2)
SSD	0.4932 (3)	1.2025 (3)	0.4101 (3)
ASSD (0.06)	0.8373 (5)	1.4218 (5)	0.5889 (5)

Panel B: n =17280 (60 days)

Daily

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.0012 (3)	0.0380 (5)	0.0314 (3)
FSD	0.0007 (1)	0.0299 (1)	0.0226 (2)
AFSD (0.06)	0.0013 (4)	0.0368 (2)	0.0365 (4)
SSD	0.0008 (2)	0.0372 (4)	0.0210 (1)
ASSD (0.06)	0.0019 (5)	0.0370 (3)	0.0501 (5)

Weekly

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.0081 (3)	0.0974 (5)	0.0834 (3)
FSD	0.0046 (1)	0.0761 (1)	0.0600 (2)
AFSD (0.06)	0.0085 (4)	0.0887 (2)	0.0963 (4)
SSD	0.0050 (2)	0.0933 (3)	0.0536 (1)
ASSD (0.06)	0.0126 (5)	0.0938 (4)	0.1346 (5)

Monthly

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.0365 (4)	0.2153 (5)	0.1698 (3)
FSD	0.0207 (1)	0.1701 (1)	0.1218 (2)
AFSD (0.06)	0.0360 (3)	0.1794 (2)	0.2006 (4)
SSD	0.0209 (2)	0.1867 (3)	0.1118 (1)
ASSD (0.06)	0.0548 (5)	0.1985 (4)	0.2761 (5)

Quarterly

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.1655 (4)	0.5973 (5)	0.2771 (3)
FSD	0.0819 (2)	0.4183 (2)	0.1958 (1)
AFSD (0.06)	0.1306 (3)	0.4308 (4)	0.3032 (4)
SSD	0.0702 (1)	0.3521 (1)	0.1994 (2)
ASSD (0.06)	0.1845 (5)	0.4246 (3)	0.4345 (5)

Yearly

Strategy	$AV(\boldsymbol{PER})$	$SD(\boldsymbol{PER})$	$SR(\boldsymbol{PER})$
BH	0.7676 (5)	1.4064 (5)	0.5458 (4)
FSD	0.4196 (2)	1.3153 (3)	0.3190 (2)
AFSD (0.06)	0.6309 (3)	1.2408 (2)	0.5085 (3)
SSD	0.3998 (1)	1.3586 (4)	0.2943 (1)
ASSD (0.06)	0.6623 (4)	0.6145 (1)	1.0778 (5)

Note: This table reports the average values, standard deviations, and Sharpe ratios of the periodic excess returns among buy-and-hold (BH) strategy, FSD strategy, AFSD (0.06) strategy, SSD strategy, and ASSD (0.06) strategy across daily, weekly, monthly, quarterly, yearly frequencies. The numbers in parentheses represent ascending order for estimations.. Panel A (B) reports the results for the window size of subsample return vector n =8640 (17280). T represents the total evaluation period (from January 1, 2018, 00:00:00 to December 1, 2023, 23:55:00) and is divided into J sub-periods. The jth sub-period consists of T_j time points, and thus $T = \sum_{j=1}^{J} T_j$. The sub-periods of one day, one week, one month, one quarter, or one year are considered. The periodic excess return for the jth sub-period is calculated as $PER_j = \prod_{i=1}^{T_j}(1 + ER_i) - 1$, and the average value, standard deviation, and Sharpe ratio of $\boldsymbol{PER} = \{PER_j \mid j = 1, 2, 3, \ldots, J\}$ are calculated by $EV(\boldsymbol{PER}) = \sum_{j=1}^{J} PER_j / J$, $SD(\boldsymbol{PER}) = \left(\sum_{j=1}^{J}\left(PER_j - EV(\boldsymbol{PER})\right)^2/(J-1)\right)^{1/2}$, and $SR(\boldsymbol{PER}) = EV(\boldsymbol{PER})/SD(\boldsymbol{PER})$.

Table 4: Periods of bull and bear markets

Period	Market
2018-01-01 00:00:00 / 2019-01-31 00:23:55	Bear
2019-02-01 00:00:00 / 2019-07-31 00:23:55	Bull
2019-08-01 00:00:00 / 2019-12-31 00:23:55	Bear
2020-01-01 00:00:00 / 2020-02-29 00:23:55	Bull
2020-03-01 00:00:00 / 2020-03-31 00:23:55	Bear
2020-04-01 00:00:00 / 2021-04-30 00:23:55	Bull
2021-05-01 00:00:00 / 2021-07-31 00:23:55	Bear
2021-08-01 00:00:00 / 2021-11-30 00:23:55	Bull
2021-12-01 00:00:00 / 2022-12-31 00:23:55	Bear
2023-01-01 00:00:00 / 2023-12-31 23:55:00	Bull

Table 5: Strategy performances across bull and bear markets

Panel A: n =8640 (30 days)

Bull market

Strategy	$AV(\boldsymbol{AER})$	$SD(\boldsymbol{AER})$	$SR(\boldsymbol{AER})$
BH	3.7645 (5)	2.8129 (4)	1.3383 (5)
FSD	-0.2222 (1)	0.9182 (2)	-0.2420 (1)
AFSD (0.06)	1.8099 (4)	4.2218 (5)	0.4287 (3)
SSD	-0.0556 (2)	0.7001 (1)	-0.0794 (2)
ASSD (0.06)	1.1454 (3)	2.0506 (3)	0.5586 (4)

Bear market

Strategy	$AV(\boldsymbol{AER})$	$SD(\boldsymbol{AER})$	$SR(\boldsymbol{AER})$
BH	-0.7312 (1)	0.1516 (1)	-4.8223 (1)
FSD	-0.0755 (3)	0.5431 (3)	-0.1390 (3)
AFSD (0.06)	-0.5206 (2)	0.2159 (2)	-2.4114 (2)
SSD	0.7153 (4)	1.3250 (4)	0.5398 (4)
ASSD (0.06)	2.5268 (5)	3.5301 (5)	0.7158 (5)

Panel B: n =17280 (60 days)

Bull market

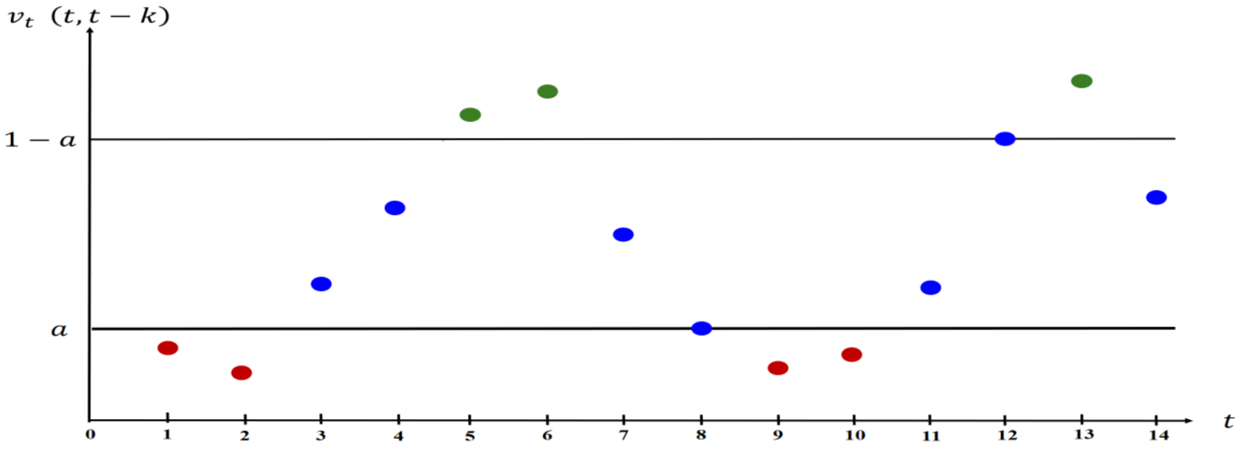
Strategy	$AV(\boldsymbol{AER})$	$SD(\boldsymbol{AER})$	$SR(\boldsymbol{AER})$
BH	3.7645 (5)	2.8129 (4)	1.3383 (5)
FSD	1.3570 (3)	1.7810 (3)	0.7619 (4)
AFSD (0.06)	2.3005 (4)	3.9905 (5)	0.5765 (2)
SSD	-0.3504 (1)	0.4521 (1)	-0.7751 (1)
ASSD (0.06)	0.4312 (2)	0.6777 (2)	0.6363 (3)

Bear market

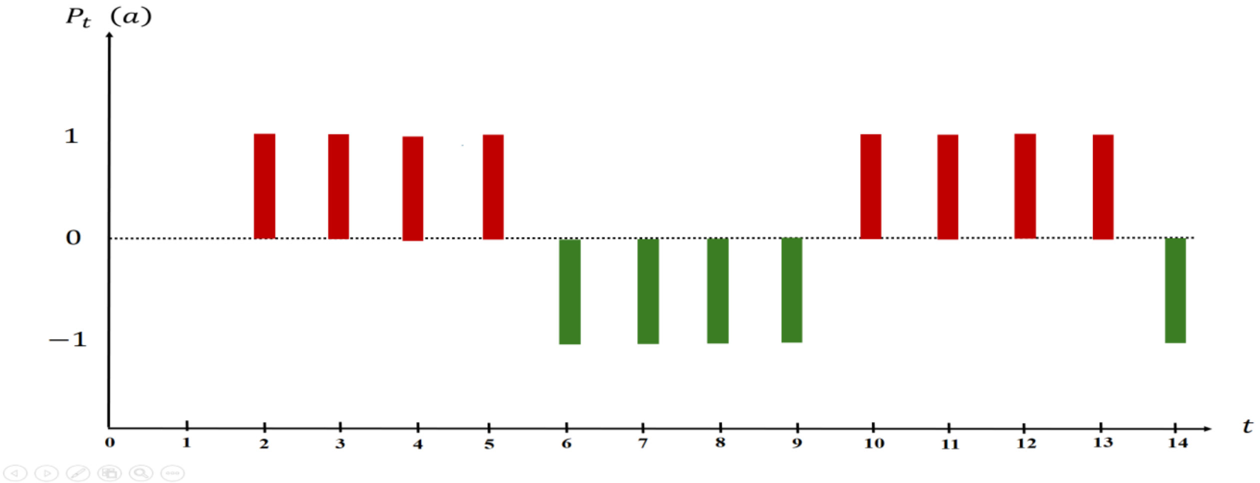
Strategy	$AV(\boldsymbol{AER})$	$SD(\boldsymbol{AER})$	$SR(\boldsymbol{AER})$
BH	-0.7312 (1)	0.1516 (1)	-4.8223 (1)
FSD	-0.1715 (2)	0.6579 (2)	-0.2606 (2)
AFSD (0.06)	0.3405 (3)	0.7450 (3)	0.4570 (3)
SSD	1.2671 (5)	1.9709 (5)	0.6429 (4)
ASSD (0.06)	1.2069 (4)	0.9108 (4)	1.3251 (5)

Note: This table reports the average values, standard deviations, and Sharpe ratios of the excess returns among buy-and-hold (BH) strategy, FSD strategy, AFSD (0.06) strategy, SSD strategy, and ASSD (0.06) strategy in bull and bear markets. The numbers in parentheses represent ascending order for estimations. Panel A (B) reports the results for the window size of subsample return vector n =8640 (17280). T represents the total evaluation period (from January 1, 2018, 00:00:00 to December 1, 2023, 23:55:00). J^{Bull} (J^{Bear}) denotes the numbers of the bull (bear) market periods, and T_j^{Bull} (T_j^{Bear}) denotes the numbers of 5-min excess returns generated by each strategy in jth bull (bear) market periods. Thus, $T=\sum_{j=1}^{J^{Bull}} T_j^{Bull}+\sum_{j=1}^{J^{Bear}} T_j^{Bear}$. Let $M \in \{Bull, Bear\}$, and jth period excess return in bull or bear market is calculated as $PER_j = \prod_{i=1}^{T_j^M}(1+ER_i)-1$. The annualized period excess return for PER_j is calculated as $AER_j = \left(1+PER_j\right)^{1/Y_j^M}-1$, where Y_j^M is the corresponding number of year for jth bull or bear market periods. The average value, standard deviation, and Sharpe ratio of $\boldsymbol{AER}=\left\{AER_j \mid j=1,2,3,\dots,J^M\right\}$ are calculated by $EV(\boldsymbol{AER})=\sum_{j=1}^{J^M} AER_j / J^M$, $SD(\boldsymbol{ER})=\left(\sum_{j=1}^{J^M}\left(AER_j - EV(\boldsymbol{AER})\right)^2/(J^M-1)\right)^{1/2}$, and $SR(\boldsymbol{AER})=EV(\boldsymbol{AER})/SD(\boldsymbol{AER})$.

Figures



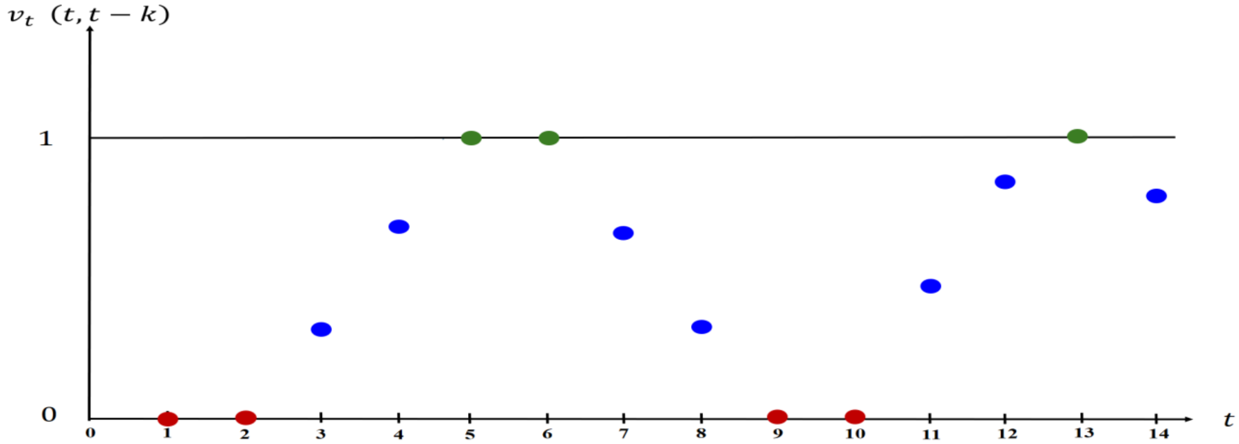
(A) Violation ratio



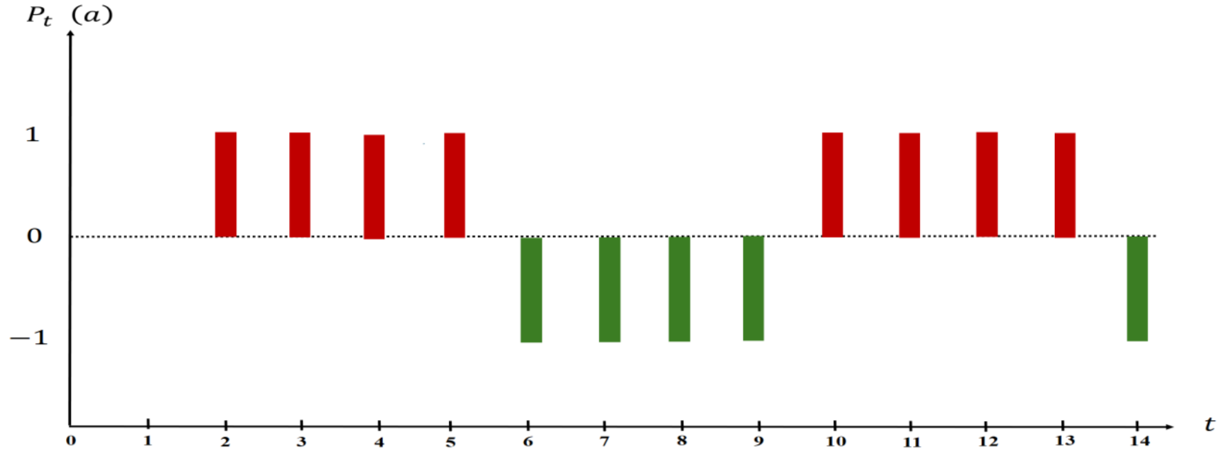
(B) Position

Figure 1. Violation ratio and position of ASD strategy.

Note: This figure provides an example of determining positions $P_t\ (a)$ through violation ratio $v_t\ (t, t-k)$ at time t given a predetermined allowance violation ratio a and lag period k based on ASD trading rules (i), (ii), and (iii) in section 2.2.1. The assumed violation ratios are shown in the top figure, while the corresponding positions are displayed in the bottom figure. We assume that $P_0\ (a) = 0$. At $t = 1$ and 2, since $v_1\ (1, 1-k)$ and $v_2\ (2, 2-k)$ are both smaller than a, we have $P_2\ (a) = P_3\ (a) = 1$ based on ASD trading rule (i). At $t = 3$ and 4, because both $v_3\ (3, 3-k)$ and $v_4\ (4, 4-k)$ are between a and $1-a$ and $P_3\ (a) = 1$, ASD trading rule (iii) indicates that $P_4\ (a) = P_5\ (a) = 1$. Furthermore, At $t = 5$ and 6, $v_5\ (5, 5-k)$ and $v_6\ (6, 6-k)$ are both larger than $1-a$, so ASD trading rule (ii) generates $P_6\ (a) = P_7\ (a) = -1$. Because $a \leq v_7\ (7, 7-k) \leq 1-a$, ASD trading rule (iii) generates $P_8\ (a) = -1$, and similarly, because $a \leq v_8\ (8, 8-k) \leq 1-a$, ASD trading rule (iii) also generates $P_9\ (a) = -1$. At $t = 9$ and 10, both $v_9\ (9{,}9-k)$ and $v_{10}(10, 10-k)$ are smaller than a, and thus $\mathrm{P}_{10}(\mathrm{a}) = \mathrm{P}_{11}(\mathrm{a}) = 1$ based on ASD trading rule (i). Because $P_{11}(a) = 1$ and both $v_{11}(11, 11-k)$ and $v_{12}(12, 12-k)$ are between a and $(1-a)$, $P_{12}(a) = P_{13}(a) = -1$ based on ASD trading rule (iii). Final, according to ASD trading rule (ii), $P_{14}(a) = 1$ because $v_{13}(13, 13-k) > a$.



(A) Violation ratio



(B) Position

Figure 2. Violation ratio and position of SD strategy.

Note: This figure provides an example of determining positions $P_t\ (a)$ through violation ratio $v_t\ (t, t-k)$ at time t given lag period k based on SD trading rules (i), (ii), and (iii) in section 2.2.2. The assumed violation ratios are shown in the top figure, while the corresponding positions are displayed in the bottom figure. We assume that $P_0\ (0) = 0$. At $t = 1$ and 2, since $v_1\ (1, 1-k)$ and $v_2\ (2, 2-k)$ are both equal to 0, we have $P_2\ (0) = P_3\ (0) = 1$ based on SD trading rule (i). At $t = 3$ and 4, because $0 < v_3\ (3, 3-k) < 1$ and $0 < v_4\ (4, 4-k) < 1$ and $P_3\ (0) = 1$, SD trading rule (iii) indicates that $P_4\ (0) = P_5\ (0) = 1$. Furthermore, At $t = 5$ and 6, both $v_5\ (5, 5-k)$ and $v_6\ (6, 6-k)$ equal 1, so SD trading rule (ii) generates $P_6\ (0) = P_7\ (0) = -1$. Because $0 < v_7\ (7, 7-k) < 1$, SD trading rule (iii) generates $P_8\ (0) = -1$, and similarly, because $0 < v_8\ (8, 8-k) < 1$, SD trading rule (iii) also generates $P_9\ (0) = -1$. At $t = 9$ and 10, both $v_9\ (9{,}9-k)$ and $v_{10}(10, 10-k)$ equal 0, and thus $\mathrm{P}_{10}(0) = \mathrm{P}_{11}(0) = 1$ based on SD trading rule (i). Because $P_{11}(0) = 1$ and $0 < v_{11}(11, 11-k) < 1$ and $0 < v_{12}(12, 12-k) < 1$, $P_{12}(0) = P_{13}(0) = -1$ based on SD trading rule (iii). Final, according to SD trading rule (ii), $P_{14}(0) = 1$ because $v_{13}(13, 13-k) = 1$.